Right Triangles and Trigonometry

Geometry

Chapter 9

- This Slideshow was developed to accompany the textbook
  - *Big Ideas Geometry*
  - *By Larson and Boswell*
  - *2022 K12 (National Geographic/Cengage)*
- Some examples and diagrams are taken from the textbook.

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9.1 The Pythagorean Theorem

After this lesson…

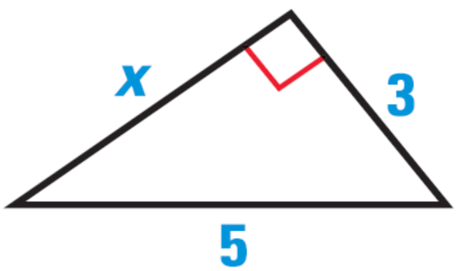
- I can list common Pythagorean triples.
- I can find missing side lengths of right triangles.
- I can classify a triangle as *acute*, *right*, or *obtuse* given its side lengths.

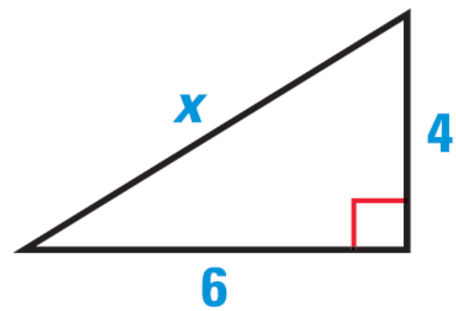
9.1 The Pythagorean Theorem

Pythagorean Theorem

In a right triangle, $\mathbf{a^2 + b^2 = c^2}$ where **a and b** are the length of the **legs** and **c** is the length of the **hypotenuse**.

- Find the value of *x*





- Try #2

$$3^2 + x^2 = 5^2$$
$$9 + x^2 = 25$$
$$x^2 = 16$$
$$x = 4$$

$$6^2 + 4^2 = x^2$$
$$36 + 16 = x^2$$
$$52 = x^2$$
$$x = 2\sqrt{13}$$

9.1 The Pythagorean Theorem

- Pythagorean Triples
  - A set of three positive integers that satisfy the Pythagorean Theorem

3, 4, 5	**5, 12, 13**	**8, 15, 17**	**7, 24, 25**
6, 8, 10	10, 24, 26	16, 30, 34	14, 48, 50
9, 12, 15	15, 36, 39	24, 45, 51	21, 72, 75
30, 40, 50	50, 120, 130	80, 150, 170	70, 240, 250
$3x, 4x, 5x$	$5x, 12x, 13x$	$8x, 15x, 17x$	$7x, 24x, 25x$

9.1 The Pythagorean Theorem

Converse of the Pythagorean Theorem

If $\mathbf{a^2 + b^2 = c^2}$ where **a and b** are the length of the short sides and **c** is the length of the **longest side**, then it is a right triangle.

- Tell whether a triangle with the given sides is a right triangle.
- $4, 4\sqrt{3}, 8$
- Try #10

$$4^2 + \left(4\sqrt{3}\right)^2 = 8^2$$
$$16 + (16)(3) = 64$$
$$16 + 48 = 64$$
$$64 = 64$$

Yes

9.1 The Pythagorean Theorem

If c is the longest side and…

$c^2 < a^2 + b^2$ → acute triangle

$c^2 = a^2 + b^2$ → right triangle

$c^2 > a^2 + b^2$ → obtuse triangle

- Show that the segments with lengths 3, 4, and 6 can form a triangle
- Classify the triangle as acute, right or obtuse.
- Try #16

$$3 + 4 > 6$$
$$7 > 6$$

$$3^2 + 4^2 \underline{?}\ 6^2$$
$$9 + 16 \underline{?}\ 36$$
$$25 < 36$$

obtuse

9.2 Special Right Triangles

After this lesson…

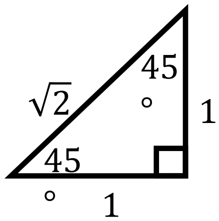
- I can find side lengths in 45°-45°-90° triangles.
- I can find side lengths in 30°-60°-90° triangles.
- I can use special right triangles to solve real-life problems.

9.2 Special Right Triangles

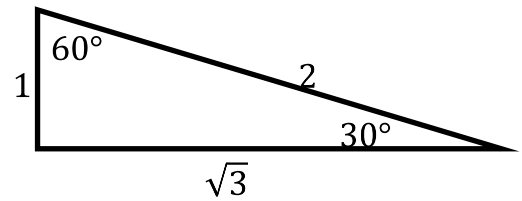
Some triangles have special lengths of sides, thus in life you see these triangles often such as in construction.

9.2 Special Right Triangles

- 45°-45°-90°



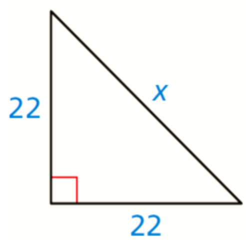
- 30°-60°-90°

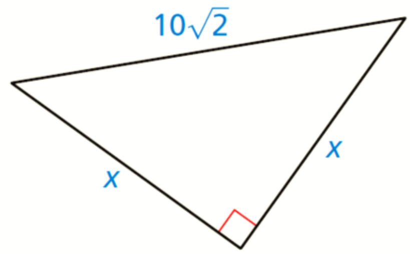


If you have another 45°-45°-90° or 30°-60°-90° triangle, then use the fact that they are similar and use the proportional sides.

9.2 Special Right Triangles

- Find the value of x. Write your answer in simplest form.





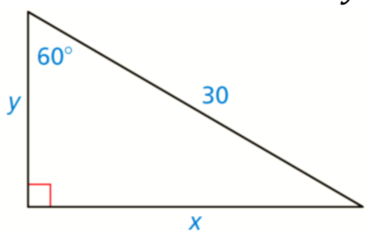
- Try #2

$$\frac{x}{22} = \frac{\sqrt{2}}{1}$$
$$x = 22\sqrt{2}$$

$$\frac{x}{10\sqrt{2}} = \frac{1}{\sqrt{2}}$$
$$x\sqrt{2} = 10\sqrt{2}$$
$$x = 10$$

9.2 Special Right Triangles

- Find the values of *x* and *y*. Write your answers in simplest form.



- Try #6

$$\frac{x}{30} = \frac{\sqrt{3}}{2}$$
$$2x = 30\sqrt{3}$$
$$x = 15\sqrt{3}$$

$$\frac{y}{30} = \frac{1}{2}$$
$$2y = 30$$
$$y = 15$$

9.3 Similar Right Triangles

After this lesson…

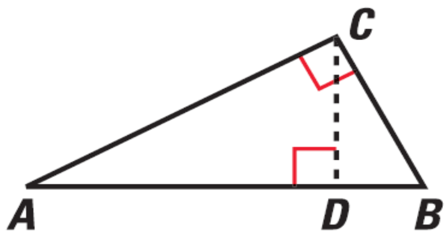
- I can explain the Right Triangle Similarity Theorem.
- I can find the geometric mean of two numbers.
- I can find missing dimensions in right triangles.

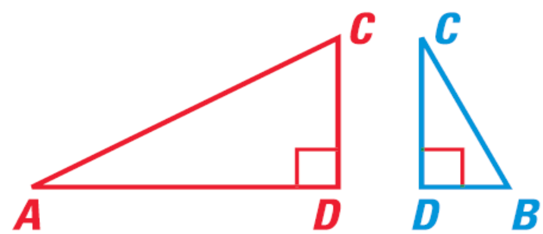
9.3 Similar Right Triangles

Right Triangle Similarity Theorem

If the altitude is drawn to the hypotenuse of a right triangle, then the two triangles formed are similar to the original triangle and to each other.

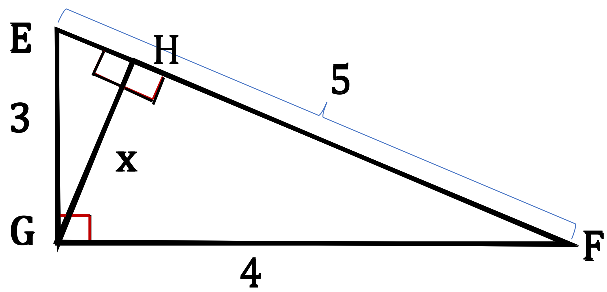
- $\Delta CBD \sim \Delta ABC$, $\Delta ACD \sim \Delta ABC$, $\Delta CBD \sim \Delta ACD$





9.3 Similar Right Triangles

- Identify the similar triangles. Then find *x*.



- Try #4

ΔEFG ~ ΔGFH ~ ΔEHG

$$\frac{GH}{EG} = \frac{GF}{EF}$$
$$\frac{x}{3} = \frac{4}{5}$$
$$x = \frac{12}{5}$$

9.3 Similar Right Triangles

Geometric Mean

The geometric mean of two positive numbers a and b is the positive number that satisfies $\frac{a}{x} = \frac{x}{b}$. So, $x = \sqrt{ab}$

- Find the geometric mean of 8 and 10.

- Try #10

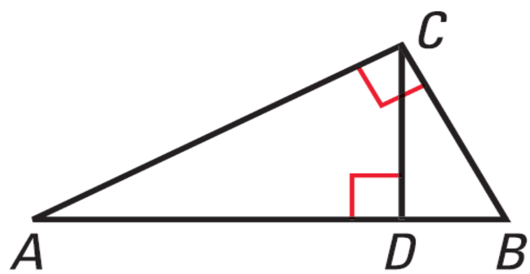
$$\sqrt{8 \cdot 10} = \sqrt{80} = 4\sqrt{5} \approx 8.9$$

9.3 Similar Right Triangles

Geometric Mean (Altitude) Theorem

If the altitude is drawn to the hypotenuse of a right triangle, then the altitude is the geometric mean of the two segments of the hypotenuse.

- $CD = \sqrt{AD \cdot DB}$

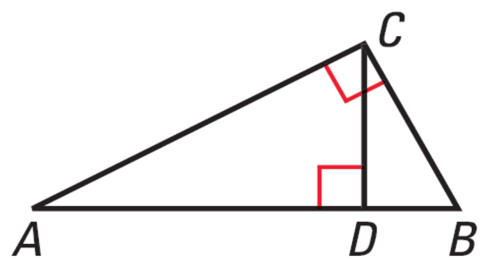


9.3 Similar Right Triangles

Geometric Mean (Leg) Theorem

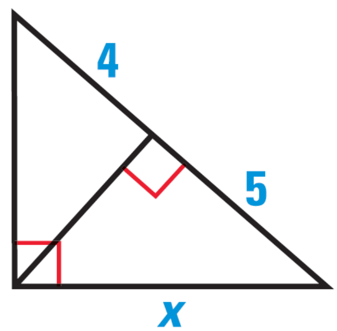
If the altitude is drawn to the hypotenuse of a right triangle, then each leg is the geometric mean of the hypotenuse and the segment of the hypotenuse adjacent to that leg.

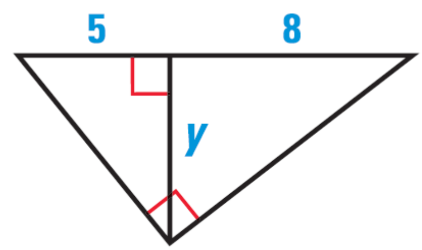
- $AC = \sqrt{AB \cdot AD}$
- $BC = \sqrt{AB \cdot DB}$



9.3 Similar Right Triangles

- Find the value of *x* or *y*.





- Try #18

$$\frac{x}{9} = \frac{5}{x}$$
$$x^2 = 45$$
$$x = 3\sqrt{5} = 6.708$$

$$\frac{y}{5} = \frac{8}{y}$$
$$y^2 = 40$$
$$y = 2\sqrt{10} = 6.325$$

9.4 The Tangent Ratio

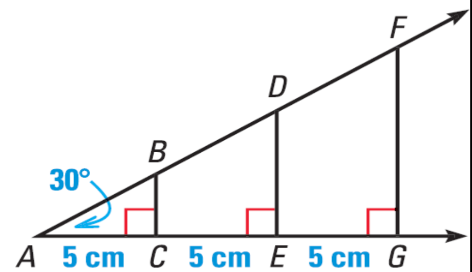
After this lesson…

- I can explain the tangent ratio.
- I can find tangent ratios.
- I can use tangent ratios to solve real-life problems.

9.4 The Tangent Ratio

- Draw a large 30° angle.
- On one side, draw a perpendicular lines every 5 cm.
- Fill in the table

Triangle	Adjacent leg	Opposite leg	$\frac{\text{Opposite leg}}{\text{Adjacent leg}}$
$\triangle ABC$	5 cm	?	?
$\triangle ADE$	10 cm	?	?
$\triangle AFG$	15 cm	?	?

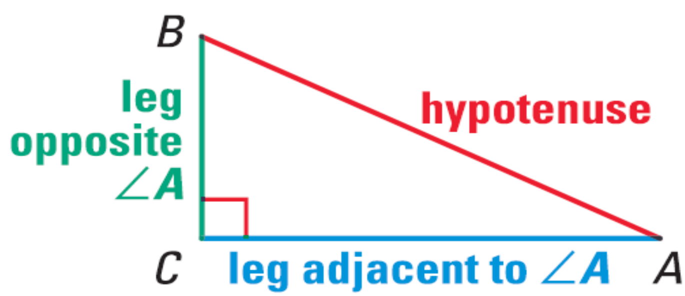


- Why are $\frac{BC}{DE} = \frac{AC}{AE}$ and $\frac{BC}{AC} = \frac{DE}{AE}$?

The triangles are similar by AA similarity

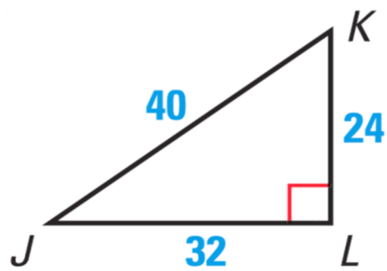
9.4 The Tangent Ratio

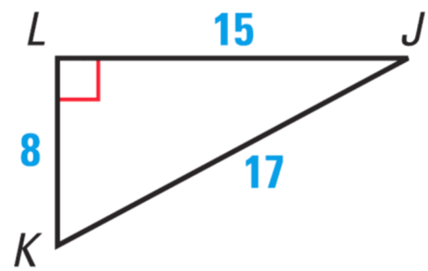
- Tangent ratio
- $\tan A = \dfrac{\text{opposite leg}}{\text{adjacent leg}}$



9.4 The Tangent Ratio

- Find tan *J* and tan *K*.





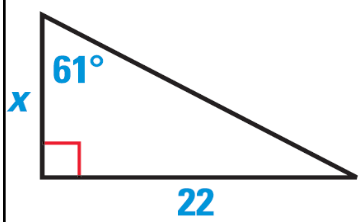
- Try #2

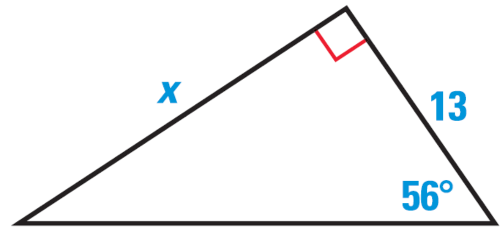
$$\tan J = \frac{24}{32} = \frac{3}{4}$$
$$\tan K = \frac{32}{24} = \frac{4}{3}$$

$$\tan J = \frac{8}{15}$$
$$\tan K = \frac{15}{8}$$

9.4 The Tangent Ratio

- Find the value of *x*. Round to the nearest tenth.





- Try #6

$$\tan 61° = \frac{22}{x}$$
$$x \tan 61° = 22$$
$$x = \frac{22}{\tan 61°} = 12.2$$

$$\tan 56° = \frac{x}{13}$$
$$13 \tan 56° = x = 19.3$$

9.5 The Sine and Cosine Ratios

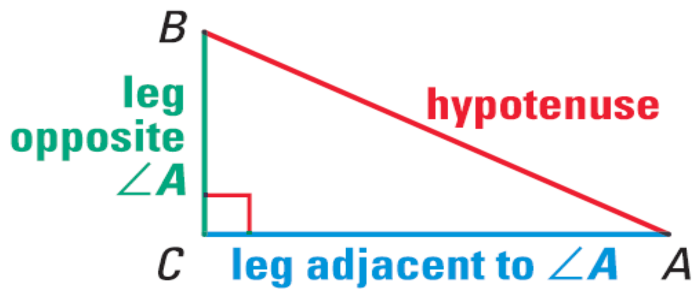
After this lesson…

- I can explain the sine and cosine ratios.
- I can find sine and cosine ratios.
- I can use sine and cosine ratios to solve real-life problems.

9.5 The Sine and Cosine Ratios

- $\sin A = \dfrac{\text{opposite leg}}{\text{hypotenuse}}$
- $\cos A = \dfrac{\text{adjacent leg}}{\text{hypotenuse}}$
- $\tan A = \dfrac{\text{opposite leg}}{\text{adjacent leg}}$

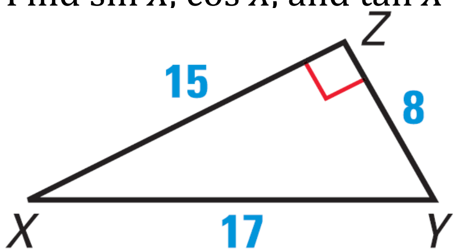
S O H
C A H
T O A



SOH = Sine Opposite Hypotenuse
CAH = Cosine Adjacent Hypotenuse
TOA = Tangent Opposite Adjacent

9.5 The Sine and Cosine Ratios

- Find sin X, cos X, and tan X

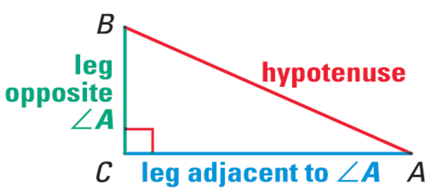


- Try #2

$$\sin X = \frac{8}{17}$$
$$\cos X = \frac{15}{17}$$
$$\tan X = \frac{8}{15}$$

9.5 The Sine and Cosine Ratios

- Sine of an angle = cosine of the complement
- $\sin A = \cos(90° - A) = \cos B$
- $\cos A = \sin(90° - A) = \sin B$

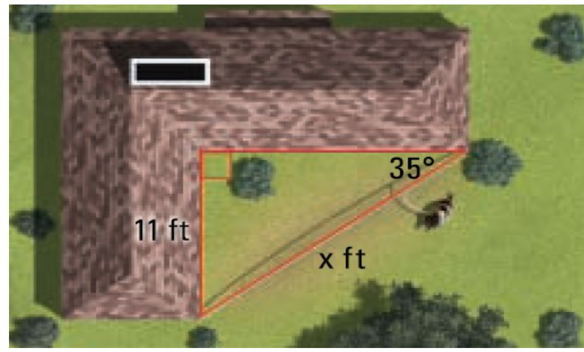


- Write cos 68° in terms of sine.
- Try #8

$\cos 68° = \sin(90° - 68°) = \sin 22°$

9.5 The Sine and Cosine Ratios

- Find the length of the dog run (x).

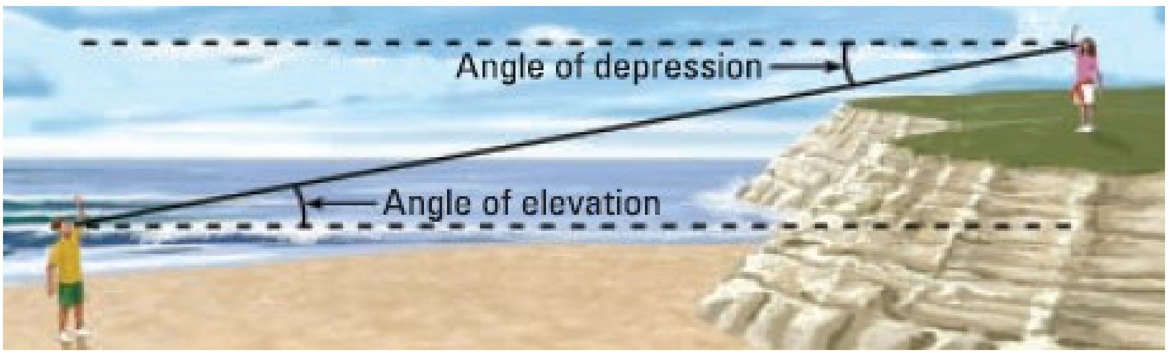


- Try #16

$$\sin 35° = \frac{11}{x}$$
$$x \cdot \sin 35° = 11$$
$$x = \frac{11}{\sin 35°} = 19.2\ ft$$

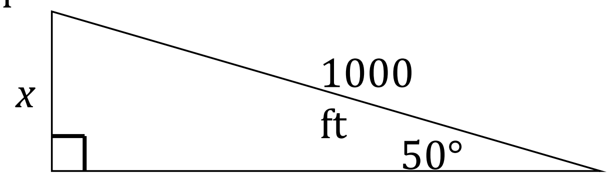
9.5 The Sine and Cosine Ratios

- Angle of Elevation and Depression
  - Both are measured from the horizontal
  - Since they are measured to ∥ lines, they are ≅



9.5 The Sine and Cosine Ratios

- The angle of elevation of a plane as seen from the airport is 50°. If the plane is 1000 ft away, how high is plane?



- Try #28

$$\sin 50° = \frac{x}{1000}$$
$$1000 \cdot \sin 50° = x$$
$$x = 766ft$$

9.6 Solving Right Triangles

After this lesson…

- I can explain inverse trigonometric ratios.
- I can use inverse trigonometric ratios to approximate angle measures.
- I can solve right triangles.
- I can solve real-life problems by solving right triangles.

9.6 Solving Right Triangles

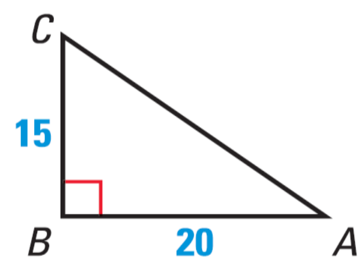
- Solve a triangle means to find all the unknown angles and sides.
  - Can be done for a right triangle if you know
    - 2 sides
    - 1 side and 1 acute angle
  - Use sin, cos, tan, Pythagorean Theorem, and Angle Sum Theorem

9.6 Solving Right Triangles

- Inverse Trigonometric Ratios
  - Used to find measures of angles when you know the sides.
  - $\sin^{-1}\frac{opp}{hyp} = \theta$
  - $\cos^{-1}\frac{adj}{hyp} = \theta$
  - $\tan^{-1}\frac{opp}{adj} = \theta$

9.6 Solving Right Triangles

- Find $m\angle D$ to the nearest tenth if $\sin D = 0.54$
- Find $m\angle C$ to the nearest tenth.



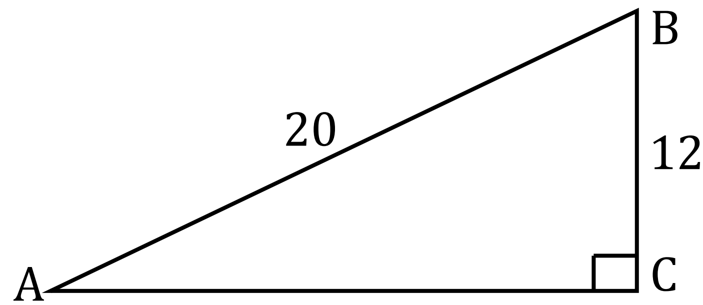
- Try #6

$$D = \sin^{-1} 0.54 = 32.7$$

$$C = \tan^{-1} \frac{20}{15} = 53.1$$

9.6 Solving Right Triangles

- Solve a right triangle that has a 12-inch leg and a 20-inch hypotenuse.



- Try #12

$$12^2 + AC^2 = 20^2$$
$$144 + AC^2 = 400$$
$$AC^2 = 256$$
$$AC = 16$$

$$\sin A = \frac{12}{20}$$
$$A = \sin^{-1}\frac{12}{20} = 36.9°$$

$$\cos B = \frac{12}{20}$$
$$B = \cos^{-1}\frac{12}{20} = 53.1°$$

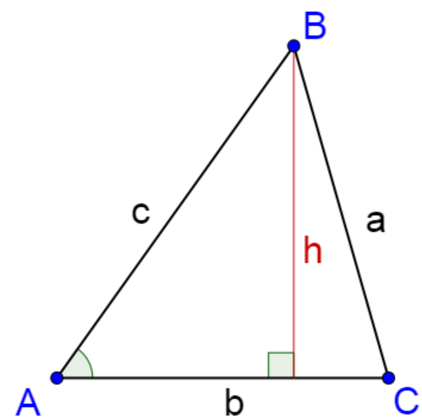
9.7A Law of Sines

After this lesson…

- I can find areas of triangles using formulas that involve sine.
- I can solve triangles using the Law of Sines.

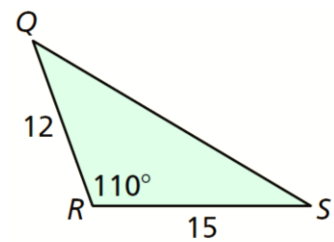
9.7A Law of Sines

- Area of a Triangle
  - $A = \frac{1}{2}bh$
    - $\sin A = \frac{h}{c}$
    - $c \sin A = h$
  - $Area = \frac{1}{2}bc \sin A$
  - $Area = \frac{1}{2}ac \sin B$
  - $Area = \frac{1}{2}ab \sin C$



9.7A Law of Sines

- Find the area of the triangle.



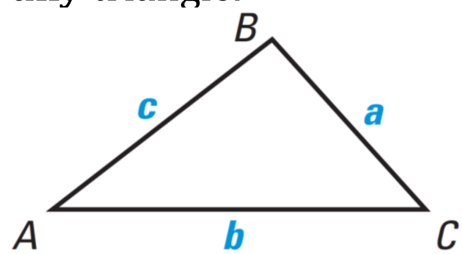
- Try #8

$$Area = \frac{1}{2} qs \sin R$$

$$Area = \frac{1}{2}(15)(12)\sin 110° \approx 84.6$$

9.7A Law of Sines

- Tangent, Sine, and Cosine are only for **right** triangles.
- Law of Sines and Law of Cosines are for **any** triangle.



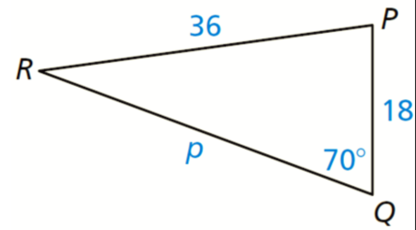
- Law of Sines
- $\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$

- Used if you know
  - AAS, ASA, SSA

Only use two of the ratios at a time.

9.7A Law of Sines

- Solve the triangle.



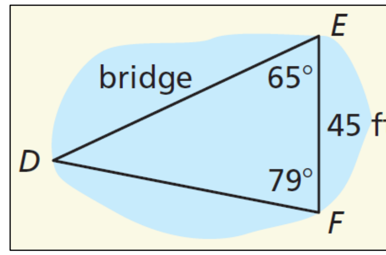
$$\frac{\sin Q}{q} = \frac{\sin R}{r}$$
$$\frac{\sin 70°}{36} = \frac{\sin R}{18}$$
$$36 \sin R = 18 \sin 70°$$
$$\sin R = 0.4698$$
$$R = \sin^{-1} 0.4698 = 28.0°$$

$$P = 180° - 70° - 28.0° = 82.0°$$

$$\frac{\sin Q}{q} = \frac{\sin P}{p}$$
$$\frac{\sin 70°}{36} = \frac{\sin 82.0°}{p}$$
$$p \sin 70° = 36 \sin 82.0°$$
$$p = 37.9$$

9.7A Law of Sines

- A surveyor makes the measurements shown to determine the length of a walking bridge to be built across a pond in a city park. Find the length of the bridge.



- Try #14

$$D = 180° - 65° - 79° = 36°$$

$$\frac{\sin D}{d} = \frac{\sin F}{f}$$
$$\frac{\sin 36°}{45} = \frac{\sin 79°}{f}$$
$$f \sin 36° = 45 \sin 79°$$
$$f = 75.2\ ft$$

$$\frac{\sin D}{d} = \frac{\sin E}{e}$$
$$\frac{\sin 36°}{45} = \frac{\sin 65°}{e}$$
$$e \sin 36° = 45 \sin 65°$$
$$e = 69.4\ ft$$

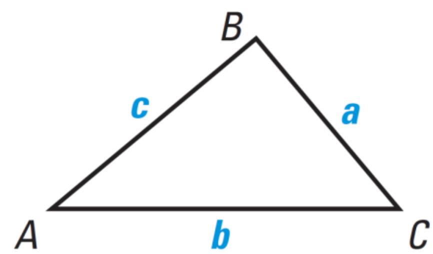
9.7B Law of Cosines

After this lesson…

- I can solve triangles using the Law of Cosines.

9.7B Law of Cosines

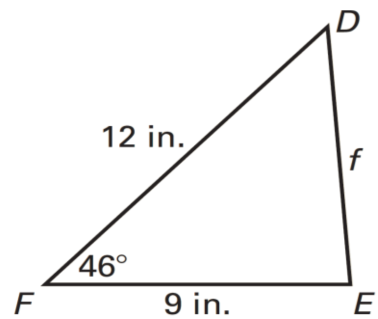
- Law of Cosines
  - $a^2 = b^2 + c^2 - 2bc \cos A$
  - $b^2 = a^2 + c^2 - 2ac \cos B$
  - $c^2 = a^2 + b^2 - 2ab \cos C$



- Use when you know
  - SSS, SAS

9.7B Law of Cosines

- Solve the triangle.



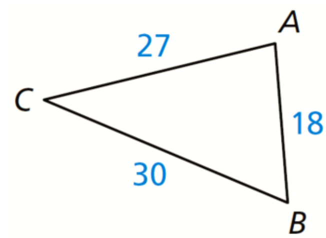
$$f^2 = d^2 + e^2 - 2de\cos F$$
$$f^2 = 9^2 + 12^2 - 2 \cdot 9 \cdot 12 \cdot \cos 46°$$
$$f^2 = 74.9538$$
$$f = 8.66\ in$$

$$d^2 = e^2 + f^2 - 2ef\cos D$$
$$9^2 = 12^2 + 8.66^2 - 2(12)(8.66)\cos D$$
$$81 = 144 + 74.9538 - 207.84\cos D$$
$$-137.9538 = -207.84\cos D$$
$$0.66375 = \cos D$$
$$D = \cos^{-1} 0.66375 \approx 48.4°$$

$$E = 180° - 46° - 48.4° = 85.6°$$

9.7B Law of Cosines

- Solve the triangle.



- Try #22

$$a^2 = b^2 + c^2 - 2bc\cos A$$
$$30^2 = 27^2 + 18^2 - 2(27)(18)\cos A$$
$$900 = 729 + 324 - 972\cos A$$
$$-153 = -972\cos A$$
$$0.1574 = \cos A$$
$$A = \cos^{-1} 0.1574 \approx 80.9°$$

$$b^2 = a^2 + c^2 - 2ac\cos B$$
$$27^2 = 30^2 + 18^2 - 2(30)(18)\cos B$$
$$729 = 900 + 324 - 1080\cos B$$
$$-495 = -1080\cos B$$
$$0.4583 = \cos B$$
$$B = \cos^{-1} 0.4583 \approx 62.7°$$

$$C = 180° - 80.9° - 62.7° = 36.4°$$